Thus, for the conditions considered here, the remainder function $R_0(k)$ can differ from unity by at most a few tenths of a percent, and within this accuracy we may use

$$f_0(k)=f_0^{(C)}(k)\exp\{-\mathrm{Tr}G_0{}^{\eta}(k)V+i\eta\psi(l+1+i\eta)\},$$

or

$$\begin{aligned}\mathfrak{F}_s&=|f_0(k)|^{-2}\\&=\mathfrak{F}_c\exp\{(\pi\eta^2\langle\mu\rangle/k)[\mathrm{cotanh}\pi\eta+1]\}.\end{aligned}\qquad(46)$$

Here $\mathfrak{F}_c$ is the Fermi function of a pure Coulomb potential,

$$\mathfrak{F}_c=|\Gamma(1+i\eta)|^2e^{-\pi\eta},\qquad(47)$$

and we have retained only terms to order μ in an expansion of the function $\mathrm{Re}\,\mathfrak{a}_0(\mu,k)$ which determines $\mathrm{ReTr}G_0{}^{\eta}V$. As an illustration of the order of the screening corrections, and to compare our results with the numerical calculation of Reitz,[11] we consider again the example of $E=200$ keV, $Z=16$. In this case[12]

$$\mathfrak{F}_s/\mathfrak{F}_c=\begin{cases}1+7\times10^{-3}(1+38\times10^{-3})\,, & \text{positron decay}\\ 1-3\times10^{-3}(1+0\times10^{-3})\,, & \text{electron decay.}\end{cases}$$

The parenthesis enclose the corresponding values found by Reitz. His value for the positron-decay correction disagrees quite strongly with ours. We also note that, to within terms of order $(\langle\mu\rangle/k)^2$, we may write our result (46) for the Fermi function as

$$\mathfrak{F}_s=(k'/k)\mathfrak{F}_c{}',\qquad(48)$$

where $\mathfrak{F}_c{}'$ is the pure Coulomb Fermi function evaluated at the shifted energy $E'=E+Z\alpha\langle\mu\rangle$, and k' is the wave number corresponding to this shifted energy. This form agrees with the WKB result of Rose.[13]

ACKNOWLEDGMENTS

I am indebted to Professor L. Durand, III, for bringing the screening-correction problem to my attention and for many useful conversations. I have enjoyed fruitful discussions with Dr. Levere Hostler.

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[11] J. R. Reitz, Phys. Rev. **77**, 10 (1950).

[12] It must be noted that these values are of the same order as that of the error bound $\|B_0(k)\|^2$. Indeed, the correction terms in (46) and $\|B_0(k)\|^2$ are both of order $\langle\mu\rangle/k$. However, an inspection of the determinantal representation of the remainder function $R_0(k)$ of Eq. (23) shows that its absolute value is *not* of order $\langle\mu\rangle/k$ as indicated by the bound $\|B_0(k)\|^2$, but rather of the smaller order $(\langle\mu\rangle/k)^2$. The reason for this discrepancy is that $\|B_0(k)\|^2$ gives essentially a bound on the logarithm of $R_0(k)$, and $R_0(k)$ has a large phase of order $\langle\mu\rangle/k$.

[13] M. E. Rose, Phys. Rev. **49**, 727 (1936).

PHYSICAL REVIEW VOLUME 135, NUMBER 2B 27 JULY 1964

Empirical Screening Correction for *M*-Subshell Internal Conversion Coefficients*

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The only theoretical values which are available for coefficients of internal conversion in the M shell have been calculated without the inclusion of screening, and they are in disagreement with experimental values by factors as large as 3. From the comparison of these theoretical values with new accurate measurements on the M-subshell electron lines of the $M4$ transitions occuring in the decay of Te^{121m} and of Te^{123m}, it was possible to effect a tentative semiempirical screening correction. Essentially, this is the replacement of the nuclear charge Z for the evaluation of the coefficient by $Z_{\mathrm{eff}M}=Z-\sigma_i$, where $\sigma_i=7.0$, 7.9, and 10.0 for M_{I} (3*s*), $M_{\mathrm{II,III}}$ (3*p*), and $M_{\mathrm{IV,V}}$ (3*d*) electrons, respectively. This correction to the theoretical values is found to produce agreement with other experimental M conversion results, both measured in this work and taken from the literature, over a wide range of multipolarities and of Z and energy values. The nonspecific characteristic of the correction is interpreted to mean that the screening is chiefly an effect on the electron wave functions of the initial bound states of the atom.

I. INTRODUCTION

It has been recognized that experimentally determined values of internal conversion coefficients in the M levels are considerably smaller than the theoretical values now available.[1] For simplicity, two effects included in the later theoretical work on K and L shell conversion coefficients[1,2] were neglected in the M-shell calculations. The first of these, the effect of finite nuclear size, was thought to be of little importance in most cases; it was recognized that the second effect, the screening of the M electrons from the nuclear charge by the other electrons in the atom could produce

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[1] M. E. Rose, *Internal Conversion Coefficients* (North-Holland Publishing Company, Amsterdam, 1958).

[2] L. A. Sliv and A. M. Band, Academy of Sciences of the U.S.S.R., *Coefficients of Internal Conversion of Gamma Radiation* (English transl.: Physics Department, University of Illinois, Urbana, Reports 57 ICC K1 and 58 ICC L1, 1957 and 1958).

appreciable corrections. In a survey of experimental data covering a wide range of energy and Z values, it was pointed out by Listengarten[3] that ratios of total M-shell conversion to total L-shell conversion scatter about the value 0.3, independent of multipole order, and that the theoretical ratio values differ from 0.3 by factors of very roughly 2. Listengarten concluded that the theoretical results are useful only for the determination of M-subshell ratios. If indeed screening is the major factor contributing to the discrepancy between the computed and measured coefficients, then even this conclusion would not be well justified, because it is not to be expected that screening effects on the $3s$, $3p$, and $3d$ electrons would be the same.

In the work here described measurements were made of K, L-subshell, and M-subshell lines produced by the 81.78-keV $M4$ transition in 154-day Te^{121m} and by the 88.46-keV $M4$ transition[4,5] in 104-day Te^{123m}. As was expected, the K/L_i ratios were found to agree with those derived from the tabulated coefficients,[1,2] and the M_i/K or M_i/L_i ratio values were found to be in disagreement. Absolute values of the M-subshell coefficients were obtained from these ratio values with the assumption that the theoretical values for the K and L shells are correct; and screening corrections for the several M subshells were deduced by comparison of these absolute values with the theoretical ones for lower values of Z. The validity and general applicability of the correction has been tested with other M-shell experimental data, some obtained in this work and some from the literature.

II. EXPERIMENTAL METHOD

A. Preparation of Sources

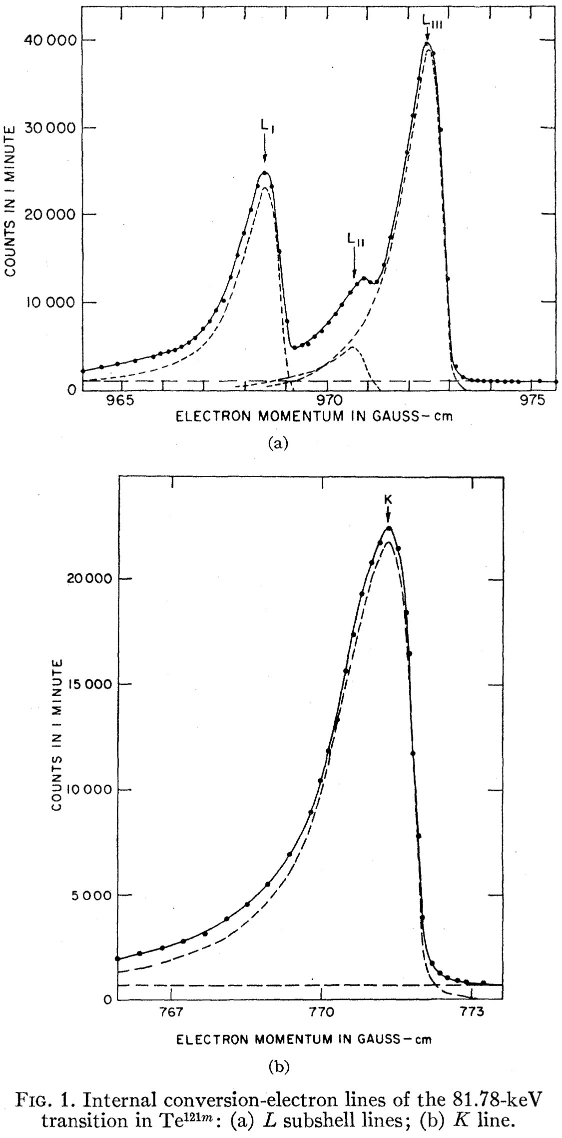
The Te^{121m} and Te^{123m} used in this work were in a source whose preparation has been described.[5] These nuclides were produced by deuteron irradiation of a natural antimony target, and they were chemically separated from it and from other possible products. For the conversion-electron spectroscopy, the Te activities were electroplated onto a gold foil in an area about 0.75 mm wide and 15 mm long.

A Ba^{137m} source was prepared by vacuum sublimation of carrier-free Cs^{137} onto a masked aluminum foil.[6]

B. The β-Ray Spectrometer and its Calibration

The instrument used in this work is the 50-cm radius $\pi\sqrt{2}$ double-focusing iron-magnet spectrometer, which has been described elsewhere.[7] Improvements in the resolution and transmission of the spectrometer have recently been made by installation of external iron shims. For electron energies above about 100 keV, momentum resolution of 0.05% full-width at half-maximum can readily be attained with source and counter slit widths of ~0.7 mm and with a transmission solid angle of 0.3%. At lower energies the peak widths are somewhat greater. Operation of the instrument has been made nearly completely automatic.

A gas-flow proportional counter fitted with a thin side window and maintained at a pressure of 25 mm of Hg with butene-2 gas was used as the detector. The window, a Mylar film lightly coated with gold, had a



(a)

(b)

FIG. 1. Internal conversion-electron lines of the 81.78-keV transition in Te^{121m}: (a) L subshell lines; (b) K line.

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[3] M. A. Listengarten, Izv. Akad. Nauk SSSR, Ser. Fiz. **22**, 759 (1958) [English transl.: Bull. Acad. Sci. USSR **22**, 755 (1958)].

[4] *Nuclear Data Sheets*, compiled by K. Way *et al.* (Printing and Publishing Office, National Academy of Sciences—National Research Council, Washington 25, D. C., 1959), NRC 60-4-81 and 60-6-69.

[5] Y. Y. Chu, O. C. Kistner, A. C. Li, S. Monaro, and M. L. Perlman, Phys. Rev. **133**, B1361 (1964).

[6] The Cs^{137} was supplied by the Isotopes Division, U. S. Atomic Energy Commission, Oak Ridge, Tennessee.

[7] G. T. Emery, W. R. Kane, M. McKeown, M. L. Perlman, and G. Scharff-Goldhaber, Phys. Rev. **129**, 2597 (1963).

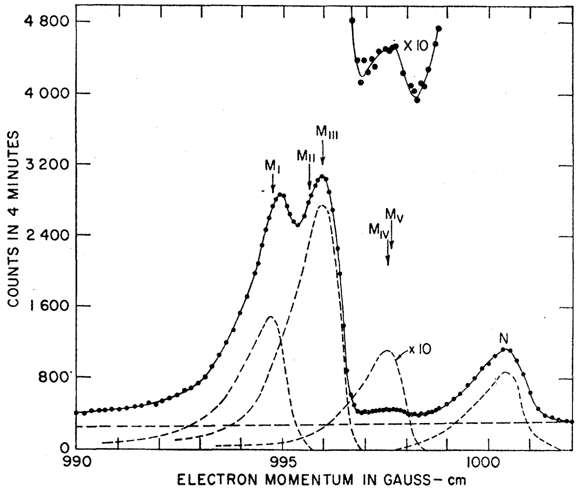

FIG. 2. *M*-shell multiplet from internal conversion of the 81.78-keV transition in Te^{121m} and the resolution of its components.

total thickness of 580 μg per cm^2. Window absorption corrections, which were necessary for electrons of energy less than 60 keV, were taken from the literature.[8]

The calibration of the spectrometer momentum scale was derived from measurements of conversion lines of the 155.032±0.012-keV transition[9] in Os^{188}.

III. RESULTS

A. The Spectra and Their Resolution

Typical conversion-electron spectra are shown in Figs. 1 and 2. In the resolution of composite lines, as for example the *L*-subshell conversion lines of the 81.78-keV[5] transition of Te^{121m} (Fig. 1a), each component was fitted to a standard line shape, taken from that of a simple line in the same spectrum. The simple line was chosen close in energy to the multiplet to be resolved so that the effects of source thickness on the line shape could be neglected. The analysis was started, usually, from the high-momentum end of the composite line. For each component, the peak height and the high-momentum side of the line were adjusted to a best fit by an approximate least squares method. The area of each subshell line thus resolved divided by its momentum was taken as a measure of the intensity.

In the case of the *M*-subshell conversion lines in the Te isomers, the relatively small differences in the binding energies made an analysis into the five individual components impracticable; it was not difficult, however, for the *M*4 transitions to effect a resolution into three components: (1) M_{I}, (2) M_{II} and M_{III} combined, and (3) M_{IV} and M_{V} combined. This analysis was done just as described above for the *L* lines, except that for $M_{\text{II,III}}$ (3*p* electrons) and for $M_{\text{IV,V}}$ (3*d* electrons) the standard line shape was taken to be that of the sum of two simple lines having M_{II}–M_{III} and M_{IV}–M_{V} spacings, respectively. Relative intensities of M_{II} and M_{III} in the $M_{\text{II,III}}$ line, and of M_{IV} and M_{V} in the $M_{\text{IV,V}}$ line, were taken from the conversion coefficients computed by Rose[1,10]; the results of the analysis are insensitive to any reasonable variation of these contribution ratios because of the smallness of the momentum separations and because of the small contribution made by M_{II} to the $M_{\text{II,III}}$ line. There are shown in Fig. 2 the measured *M* conversion lines of the 81.78-keV transition in Te^{121m} and their decomposition.

In Table I the conversion coefficients thus determined for this transition and for the analogous ones in Te^{123m} are presented. Only relative values of the coefficients were measured; values in the table have been normalized to that for the *K* line, whose coefficient has been set equal to the theoretical value.[2] It is clear that the measured *L*-subshell coefficients are in agreement with the theoretical values.

B. Extrapolation Method for Extraction of *M*-Subshell Coefficients from the Theoretical Tables

Unfortunately, the comparison of the measured and theoretical values for the *M* subshells is not so simple. Values of individual *M*-subshell coefficients[1] are avail-

TABLE I. Experimental and theoretical values of the internal conversion coefficients for the *M*4 transitions occurring in the decay of Te^{121m} and Te^{123m}. Except for the *M* subshells, the effects of screening and finite nuclear size were included in the calculation of the theoretical values.

	Te^{121m} (81.78 keV)[a]		Te^{123m} (88.46 keV)[a]	
	Conversion coefficients			
Level	Measurement[b]	Theory[c]	Measurement[b]	Theory[c]
K	[6.50(2)±0.13]	6.50(2)	[4.55(2)±0.09]	4.55(2)
L_{I}	2.91(2)±0.15	2.70(2)	1.71(2)±0.10	1.67(2)
L_{II}	6.27(1)±0.50	6.25(1)	4.21(1)±0.40	4.10(1)
L_{III}	4.86(2)±0.15	4.75(2)	2.69(2)±0.09	2.75(2)
ΣL_i	8.40(2)±0.22	8.08(2)	4.82(2)±0.14	4.83(2)
M_{I}	6.25(1)±0.50	1.33(2)	3.86(1)±0.35	7.94(1)
M_{II}	1.15(2)±0.06	2.97(1)	6.69(1)±0.35	1.89(1)
M_{III}		3.14(2)		1.82(2)
M_{IV}	4.70(0)±0.80	1.04(1)	2.98(0)±0.50	5.96(0)
M_{V}		1.49(1)		8.40(0)
ΣM_i	1.82(2)±0.07	5.02(2)	1.09(2)±0.05	2.95(2)

[a] Reference 5.

[b] Values given are normalized to that for the *K* line, the conversion coefficient of which was set equal to the theoretical value. The figure 2.91(2)±0.15 is read (2.91±0.15)×10^2.

[c] *K*- and *L*-shell theoretical values are taken from Sliv and Band (Ref. 2); *M* values are derived from the tables of Rose (Ref. 1) by a method described in the text of this paper. The uncertainties of the Sliv and Band results are stated to be 2–3%.

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[8] R. Arnoult, Ann. Phys. (Paris) **12**, 241 (1939). R. O. Lane and D. J. Zaffarano, Institute for Atomic Research and Department of Physics, Iowa State College, Ames, Iowa Report LR-211 (unpublished); published in part in Phys. Rev. **94**, 960 (1950). Also, H. Slatis, in *Beta- and Gamma-Ray Spectroscopy*, edited by K. Siegbahn (North-Holland Publishing Company, Amsterdam, 1955), p. 269.

[9] B. Lindström and I. Marklund, Arkiv Fysik **22**, 422 (1962); R. L. Graham, J. S. Geiger, R. A. Naumann, and J. M. Prospero, Can. J. Phys. **40**, 296 (1962).

[10] The extrapolation method employed to obtain this information from the tables is described in a later part of this paper.

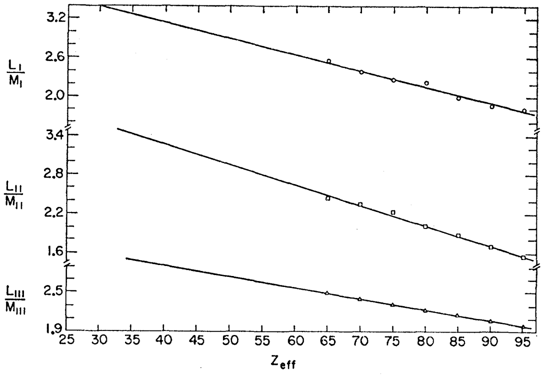

FIG. 3. Internal conversion-coefficient ratios, L_I/M_I, L_{II}/M_{II}, and L_{III}/M_{III}, for multipole order $M4$ and energy 81.78 keV as a function of Z_{eff}. The quantity Z_{eff} is $Z-\sigma$, where σ is a screening number given in the text.

able for $Z \geqslant 65$ at intervals $\Delta Z = 5$. For $Z < 65$, there are values only for the M shell in total at the same intervals. Thus, it is necessary to make not only the usual interpolation for transition energy, but also an extrapolation to the Z value 52. The method of extrapolation is based on the observation that for each of the subshells M_I, M_{II}, and M_{III} the conversion coefficient varies with Z in a manner similar to the coefficient for L_I, L_{II}, and L_{III}, respectively. Figure 3 shows the three L_i/M_i coefficient ratios for multipole-order $M4$ and energy 81.78 keV plotted as a function of Z_{eff}. In the calculation of the points shown, it was considered that the M_i and L_i values might be better comparable if in a ratio both had been calculated for the same electrostatic potential. Therefore, the Z_{eff} values for the M coefficients were taken to be equal to the Z values given by Rose; and to the tabulated L_i values, in the calculation of which Rose included the screening, there were assigned values $Z_{eff} = Z_{nuclear} - \sigma_{L_i}$, where the screening constants σ_{L_i} were taken from values in the literature.[11] By interpolation, L_i coefficients were obtained at the required integral values of Z_{eff}. Examination of Fig. 3 shows that the variation in these ratios is smooth and not great, and that extrapolation to lower Z values may be feasible. For M_{IV} and M_V conversion, one is forced to some variation of this scheme. In Fig. 4 there are shown M_{IV}/M_{III} and M_V/M_{III} conversion ratios for the same transition energy and multipolarity; these also show a smooth and rather slow variation with Z_{eff}. If the lines shown in Figs. 3 and 4 are approximately correct in the extrapolated ranges, M-subshell coefficients for $M4$ transitions of 81.78 keV may be obtained from them by use of the appropriate L_i coefficients obtained from the tabulated values. That this extrapolation method indeed seems to be useful is shown by the fact that the sum of the M-subshell conversion coefficients thus obtained for a 81.78 keV, $M4$ transition at $Z=35$, differs from the tabulated total M-shell value by only $\sim 3\%$.[12]

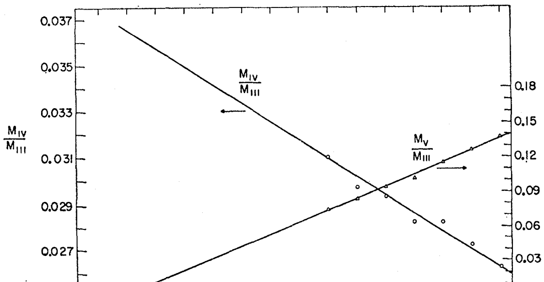

FIG. 4. Internal conversion coefficient ratios M_{IV}/M_{III} and M_V/M_{III} for multipole order $M4$ and energy 81.78 keV as a function of Z_{eff}.

C. Comparison of Measurements with Results Derived from the Rose Tables

The theoretical M-subshell coefficient values given in Table I have been obtained by this extrapolation method, and comparison of them with the measured values shows that they are too large by factors of about 2, 3, and 5 for M_I, $M_{II,III}$, and $M_{IV,V}$, respectively. It may be noted that the extrapolated theoretical values for the subshells, when summed to give totals for the M shell, agree with the total M coefficients from the Rose tables within 2%.

In the course of the work described below it became desirable to have, in addition to the information of Table I, some other M coefficient data; values for the total M shell were measured for the Ba^{137m} 661.6-keV $M4$ transition and for the Te^{123m} 159.00-keV $M1$ transition.[5] For the Ba^{137m}, the theoretical value of the K-shell coefficient was assumed in order to obtain the M-shell result by comparison of K and M conversion line intensities; in the other case no such assumption was necessary because the $M4$ transition which precedes the $M1$ in the Te^{123m} decay is essentially completely converted, and the sum of the $M4$ line intensities thus supplies the needed measure of the total $M1$ transition

TABLE II. Semiempirical screening numbers for the M subshells, derived from data on the $M4$ transitions in Te^{121m} and Te^{123m}.

Electron shell	Z_{effM} Te^{121m} 81.78 keV	 Te^{123m} 88.46 keV	$(Z_{effM})_{av}$	$\sigma_i = 52 - Z_{effM}$	σ_{BE}
M_I	45.1	44.9	45.0	7.0	10.3
$M_{II,III}$	44.2	44.1	44.1	7.9	11.6
$M_{IV,V}$	42.5	41.6	42.0	10.0	16.2

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[11] According to A. C. Douglas, D. R. Hartree, and W. A. Runciman, Proc. Cambridge Phil. Soc. **51**, 486 (1955), $\sigma_{LI} = 3.6$ and $\sigma_{LII,III} = 4.2$ for binding energy calculations in heavy elements. Although these values were actually used, it would have been simpler and, for the purposes of this work equally good, to have used $\sigma_L = 4$, or perhaps even $\sigma_L = 0$.

[12] This test is not very sensitive to errors in the M_{IV} and M_V conversion coefficients, which, for this transition, contribute very little to the total for the M shell.

TABLE III. Comparison of experimental and theoretical values of the internal conversion coefficients of a number of transitions having different multipole orders and energies. For the *M* subshells two sets of theoretical values are given, one of which includes the screening correction developed in this investigation.

Nuclide	Transition energy (keV)	Multipole order	Level	Conversion coefficients Measurement	Theory, tabulated[a]	Theory, corrected[b]
Te^{123m}	159.0	99.3% $M1$[c] 0.67% $E2$	K[d]	1.69(−1)±0.06[e]	1.63(−1)	
			L_I	2.01(−2)±0.10	1.93(−2)	
			L_{II}	1.38(−3)±0.10	1.32(−3)	
			L_{III}	3.89(−4)±0.30	4.06(−4)	
			ΣL_i	2.19(−2)±0.10	2.10(−2)	
			M_I		1.12(−2)	4.03(−3)
			M_{II}		9.28(−4)	2.48(−4)
			M_{III}		2.03(−4)	7.12(−5)
			M_{IV}		3.45(−6)	6.11(−7)
			M_V		3.48(−6)	1.08(−6)
			ΣM_i	4.54(−3)±0.23	1.23(−2)	4.35(−3)
Ba^{137m}	661.6	$M4$	K	[9.10(−2)±0.20][f]	9.10(−2)	
			L_I		1.35(−2)	
			L_{II}		1.93(−3)	
			L_{III}		1.56(−3)	
			ΣL_i	1.62(−2)±0.05	1.70(−2)	
			M_I		8.40(−3)	2.99(−3)
			M_{II}		9.37(−4)	3.72(−4)
			M_{III}		7.89(−4)	2.89(−4)
			M_{IV}		1.33(−5)	2.83(−6)
			M_V		3.45(−6)	6.72(−7)
			ΣM_i	3.54(−3)±0.12	7.14(−3)	3.66(−3)
Pu^{239}	57.26	$E2$[g]	L_I	4.1(0)±0.7	3.42(0)	
			L_{II}	[9.0(1)±1.2][f]	9.02(1)	
			L_{III}	7.6(1)±1.4	7.22(1)	
			M_I	1.4(0)±0.4	2.26(0)	8.70(−1)
			M_{II}	2.9(1)±0.5	4.13(1)	2.02(1)
			M_{III}	2.0(1)±0.3	3.61(1)	1.94(1)
			M_{IV}	3.3(−1)±0.9	7.91(−1)	3.33(−1)
			M_V	1.6(−1)±0.6	4.00(−1)	2.33(−1)
			ΣM	5.1(1)±0.6	8.09(1)	4.10(1)
Pu^{239}	67.85	$E2$[g]	L_{II}	[4.06(1)±0.6][f]	4.06(1)	
			L_{III}	2.9(1)±0.5	3.07(1)	
			M_I	4.8(−1)±1.4	1.12(0)	4.20(−1)
			M_{II}	1.0(1)±0.2	1.86(1)	9.05(0)
			M_{III}	8.2(0)±1.6	1.53(1)	8.30(0)
			M_{IV}	1.7(−1)±0.4	3.37(−1)	1.43(−1)
			M_V	7.8(−2)±2.6	1.68(−1)	9.70(−2)
			ΣM	1.9(1)±0.25	3.55(1)	1.80(1)
Pb^{205}	26.22	$M2$[h]	L_I	4.8(3)±0.4	5.30(3)	
			L_{II}	1.44(2)±0.6	2.65(2)	
			L_{III}	[2.95(3)±0.19]	2.95(3)	
			M_I	1.67(3)±0.15	2.69(3)	1.26(3)
			M_{II}	1.15(2)±0.15	1.78(2)	8.6(1)
			M_{III}	9.6(2)±1.0	1.71(3)	7.4(2)
			M_{IV}	1.6(1)±1.0	3.80(1)	1.25(1)
			ΣM	2.76(3)±0.18	4.62(3)	2.10(3)

[a] *K*- and *L*-shell coefficients in this column are from Sliv and Band (Ref. 2), who included the effects of screening and finite nuclear size in their calculations. The *M*-shell coefficients are those of Rose (Ref. 1); these are the only theoretical values now available, and screening and finite size effects were neglected in their calculation. *M*-subshell coefficients for Te^{123m} and for Ba^{137m} were obtained from the Rose tables by a method described in the text.

[b] *M*-subshell coefficients in this column have been corrected for screening by a method described in the text.

[c] Reference 5.

[d] Conversion coefficients for Te^{123m} represent absolute measurements described in the text.

[e] Read 1.69(−1)±0.06 as $(1.69\pm0.06)\times10^{-1}$.

[f] Brackets indicate that measured conversion line intensities have been normalized to an assumed value for the line so designated.

[g] Data for the Pu^{239} transitions are those of Ewan, Geiger, Graham, and MacKenzie, Phys. Rev. **116**, 950 (1959).

[h] Data for Pb^{205} are those of R. Stockendahl, Arkiv Fysik **17**, 553 (1960). We are indebted to Dr. Stockendahl for calling them to our attention.

intensity. Results of these measurements are presented in Table III of the following section.

IV. SEMIEMPIRICAL CORRECTION FOR SCREENING

In a first attempt to correct for the screening effects on internal conversion in the *M* shell, the tabulated theoretical values to be compared with the measured results were taken not at Z equal to the nuclear charge but at values $Z_{\mathrm{eff}M}=Z-\sigma_{\mathrm{BE}}$. Here σ_{BE} is a screening number, chosen separately for 3*s*, for 3*p*, and for 3*d* electrons, used in the evaluation of wave functions for the calculation of electron binding energies in moderately heavy atoms.[13] With these $Z_{\mathrm{eff}M}$ values, the

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[13] A. C. Douglas, D. R. Hartree, and W. A. Runciman, Proc. Cambridge Phil. Soc. **51**, 486 (1955)

theoretical conversion coefficients for the transitions presented in Table I are found to be considerably too small. An independent set of screening numbers σ_i was therefore evaluated by determination from the extrapolated theoretical data of the Z_{eff} values at which the theoretical conversion coefficients are equal to the measured ones. These $Z_{\text{eff}M}$ values and the σ_i values which correspond to them are presented in Table II. For comparison the σ_{BE} values are given also.

With this set of σ_i one may obtain from the tabulated theoretical values, extrapolated if necessary, M-subshell coefficients for comparison with measured values. There are shown in Table III the results of several sets of measurements of M-shell or subshell coefficients together with two sets of theoretical values, one thus corrected for screening by use of the σ_i values of Table II and the other uncorrected. Included also is information on K- and L-shell conversion of the same transitions. Some of these data have been obtained in this work; the other data have been cited from the literature and represent measurements made with counters rather than with densitometry. Several multipole orders and a range of energy and Z values are represented.

It is clear from Table III that with the use of a single set of σ_i values deduced from the data on the low-energy $M4$ transitions in the Te isomers, one obtains a very satisfactory agreement between the corrected theoretical M coefficient values and the measured ones for the transitions in the other nuclei, independent of Z, energy, and multipole order. For nuclei with $Z<30$, which have incomplete M shells, these σ_i values are, of course, not expected to apply. It should be remarked that in the Te^{123m} $M1$ and Ba^{137m} $M4$ cases, individual M-subshell lines were not separable in the measurements, and the comparison of Table III is therefore sensitive essentially only to the major contributions from M_{I}, M_{II}, and M_{III}. However, for Pu^{239} and Pb^{205} the subshell values are tested individually. Only in the Pb^{205} case is there some indication of a systematic difference between the experimental and corrected theoretical values. Whether this arises from inaccuracy of the correction at the very low energy or whether it is an error caused by normalization to L-subshell values having somewhat larger experimental uncertainties than those quoted by the authors[14] is not clear.

V. DISCUSSION

The fact that it is possible to make reasonably accurate screening corrections to the tabulated M-subshell internal conversion coefficients by use of Z values adjusted with one simple set of three screening numbers, may be interpreted qualitatively in terms of the effects on the electron wave functions for the initial states and for the continuum final states. Since the internal conversion in a given subshell of transitions of various multipolarities and energies is necessarily associated with various wave functions for the outgoing electron, it is evident either that the effects of screening on these wave functions are much the same, which is not plausible, or that the effects are small. Support for the latter argument is given by the calculations of Reitz on screening in negative beta decay.[15] Reitz showed that even for high Z (92) and low energy ($\sim$25 keV), where the effects are most pronounced, the emission probability is changed from the unscreened value by only $\sim 7\%$. Thus, the screening is mainly an effect on the initial state wave functions; and the Z independence of the σ_i values is a reflection of the fact that the K, L, and M electron wave functions scale more or less together with change of the nuclear charge.

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[14] The L-subshell discrepancy in this case is an indication that the uncertainties quoted may have been underestimated.

[15] J. R. Reitz, Phys. Rev. **77**, 10 (1950).